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HOPF BIFURCATION ANALYSIS

A Frequency Domain Approach

Jorge L Moiola

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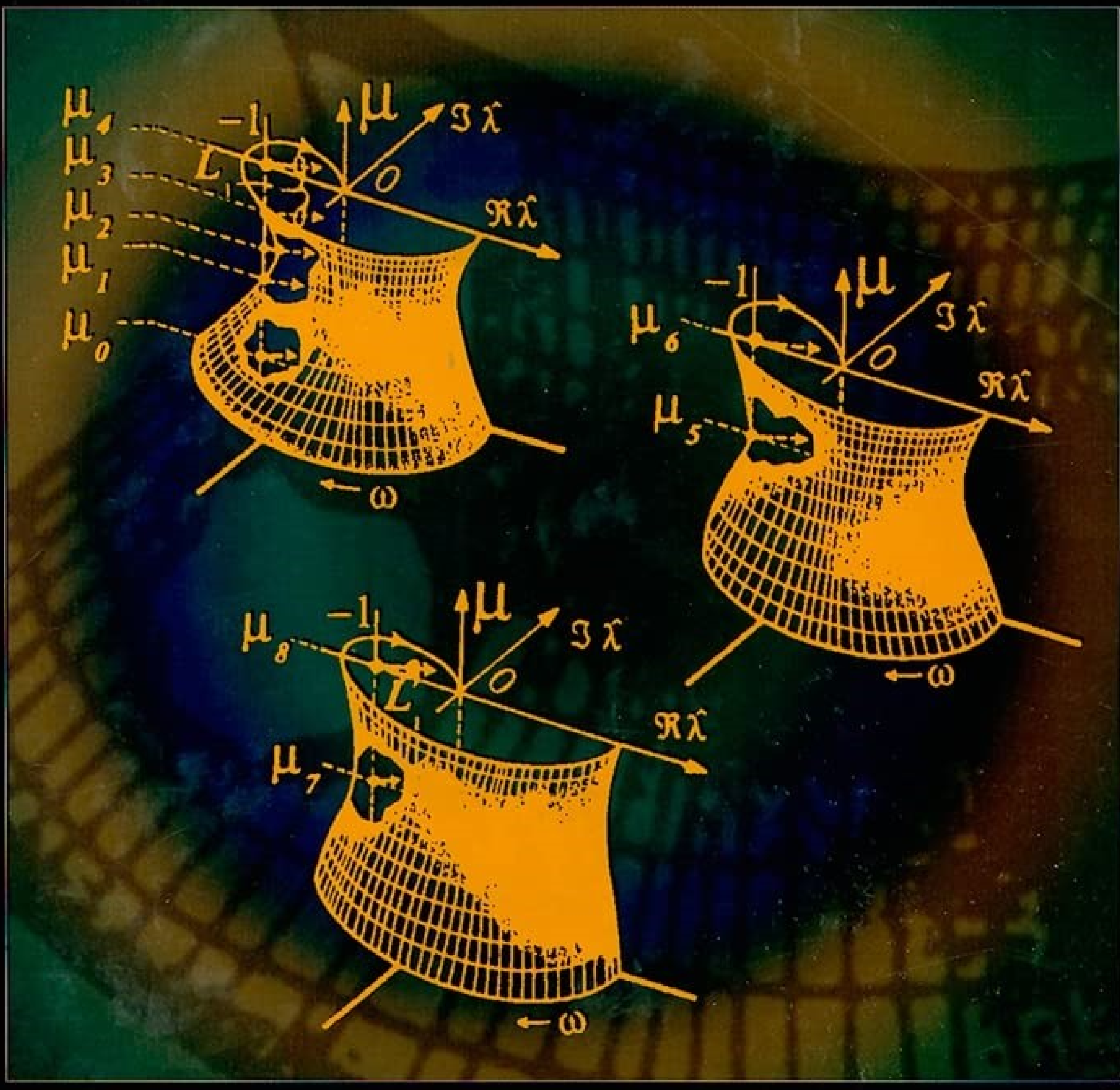

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HOPF BIFURCATION ANALYSIS — A FREQUENCY DOMAIN APPROACH

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Dedicated to the memory of

Lorenzo Bernardo Moiola (1900-1995)

and

Sheng Chen (1917-1979)

Preface

The fundamental theory of periodic solutions (in particular, limit cycles) of nonlinear ordinary differential equations (ODEs) was mainly attributed to the great French mathematician Jules Henri Poincaré (1854-1912). The idea of representing the dynamics of a nonlinear ODE in the phase-space by using what is called the Poincaré return map today, and the preliminary results of the limit cycles theory such as their existence and characteristics, are just a few pieces of the most valuable legacies that Poincaré left to the modern scientific and engineering communities. The significance and generality of his profound analysis has made an extraordinary impact on the analytical theories of nonlinear ODEs and dynamical systems, and has greatly motivated his successors in the pursuit of the modern nonlinear sciences.

For two-dimensional ODE systems, the earlier conjecture about the existence of periodic solutions given by Poincaré was formally presented by the Soviet mathematician A. A. Andronov and his colleagues. Ever since then, this existence result for periodic solutions of two-dimensional ODEs has been referred to as the Poincaré-Andronov conjecture in the literature. Independently, the German mathematician E. Hopf published an elegant result that shows the existence of limit cycles in n-dimensional ODE systems, for $n \geq 2$, assuming only the smoothness of the nonlinear vector fields of the systems. This is the celebrated Hopf Theorem. Basically, the theorem proves that the amplitude and frequency of a periodic solution of such a system can be approximately calculated when a key real parameter of the system is varied. In addition, the theorem explains how the stability of the periodic solution, which is *bifurcating* from the equilibrium can be determined as the key parameter varies. For this reason, the result is also called the Hopf bifurcation theorem. This important result was reconfirmed and applied about thirty years later by many other researchers from different disciplinary fields.

All the aforementioned works use the state-space formulation, namely, a system of nth-order ordinary differential equations. This will be referred to as the "time domain" approach in this book. Yet there is another interesting formulation of the same dynamical systems available in the literature. This alternative representation applies the familiar engineering feedback systems theory and methodology: an approach described in the "frequency domain," the complex domain after the standard Laplace transforms have been taken on the time domain state-space system. The frequency domain approach was initiated by Allwright, Mees and Chua in the late 1970's. This new methodology has an enjoyable engineering flavor and, indeed, possesses several advantages over the classical time domain methods. A typical one is its pictorial characteristic that utilizes advanced computer graphical capabilities and so bypasses quite a lot of profound and difficult mathematical analysis. As a result, it visualizes some fairly complex dynamical behavior. This book is devoted to this frequency domain approach, for both regular and degenerate Hopf bifurcation analyses.

It is perhaps important to point out at the very beginning of this book that as we proceed with thorough discussions in the following chapters, the reader will realize that many significant results and computational formulas obtained in the studies of regular and degenerate Hopf bifurcations from the time domain approach can also be translated and reformulated into the corresponding frequency domain setting, and be reconfirmed and rediscovered by using the frequency domain methods. It is also worth mentioning that some new results on other oscillatory phenomena, for example period-doubling sequences, recently appeared in the literature were developed under a frequency domain framework that is very close to the one described in this book. Looking into the near future, this stimulating and promising approach may lead to new techniques for effectively *controlling bifurcations* and *chaos.*

In this book, we will show in detail how the frequency domain approach can be used to obtain several types of standard bifurcation conditions for general nonlinear dynamical systems. We will also demonstrate a very rich pictorial gallery of local bifurcation diagrams for nonlinear systems under simultaneous variations of several system parameters. In addition, in conjunction with this graphical analysis of local bifurcation diagrams, we will present the defining and nondegeneracy conditions for several degenerate Hopf bifurcations. With a great deal of algebraic computation, we will also derive some higher-order harmonic balance approximation formulas for analyzing the dynamical behavior in small neighborhoods of certain types of degenerate Hopf bifurcations that involve multiple limit cycles and multiple limit points of limit cycles (*i.e.*, coalescences between stable and unstable limit cycles). These useful formulas enable us to better approximate the amplitude and frequency of oscillations that are "far away" from an equilibrium. In this regard, with the improved approximations we are able to describe more accurately those limit cycles that contain an important harmonic content: the amplitudes of higher-order harmonics have comparable sizes with respect to the amplitude of the first-order harmonic. Because all these topics are confronted in the current research, this book is designed and written in a style of research monographs rather than classroom textbooks. Thus, the most recent contributions to the field can be included with references.

The book is organized as follows. To prepare for the contents of the book, in Chapter 1 we first review some fundamental mathematical concepts and results of nonlinear dynamical systems in the time domain setting, which will be helpful in one's reading through the entire book.

In Chapter 2, both the time domain and frequency domain approaches to the classical Hopf bifurcation theorem will be introduced. A circuit example will be given to show that these two different versions of the regular Hopf bifurcation theorem are indeed equivalent. We then comment on the advantages as well as limitations of the frequency-domain approach. An application of the graphical (frequency domain) Hopf bifurcation theory to a chemical reaction model will also be discussed in this chapter.

Chapter 3 is devoted to a study of some explicit formulas that can be used as efficient conditions to recover the degenerate (*i.e.*, singular) bifurcation points of a dynamical system under simultaneous variations of several system parameters. A graphical method for computing certain singularities that are crucial to the understanding of the global dynamics will be developed. Some engineering applications of this method will also be discussed.

Based on the formulas for approximating the periodic solutions emerging from Hopf bifurcations, and for the continuation of certain bifurcation points in a two-dimensional parameter set that were developed in the last two chapters, we then present a methodology in Chapter 4 for computation of the local bifurcation diagrams obtained near the simplest Hopf bifurcation degeneracies. In this chapter, we first analyze the multiplicity of equilibrium solutions; then discuss multiple Hopf bifurcation points; and finally show an application of these results to the chemical reactor model discussed earlier.

Chapter 5 studies an extension of the validity domain for the periodic solutions, where the technique of higher-order harmonic balance approximations is applied under the variation of the main system parameter. The results on the continuation of periodic solutions obtained from the frequency domain approach will be verified and compared to those obtained by using the well-known and effective AUTO program software. To develop higher-order Hopf bifurcation formulas, we will describe a computational methodology that uses very high-order (the fourth, sixth and eighth order) harmonic balance approximations, for the calculation of periodic solutions of a general nonlinear dynamical system. A computational algorithm is derived in this chapter, for the continuation of periodic solutions near degenerate Hopf bifurcation points of certain types. The chapter finally offers a new method for efficiently recovering multiple limit cycles, and discusses some related applications.

In Chapter 6, we apply the frequency-domain methods developed in the previous chapters to detect oscillations in a nonlinear feedback system that contain time delays. As is well known, there will be an infinite number of eigenvalues in the corresponding linearized system, so that it can be expected that a great diversity of multiple and degenerate Hopf bifurcations exist as compared to the nonlinear systems without time delays. Two different cases, both important, will be considered: (i) systems that have time delay only in the linear part; and (ii) systems that have time delays in both the linear and the nonlinear parts.

Finally, in Chapter 7, we will develop some approximation formulas for calculation of the defining conditions (*i.e.*, stability indexes, or curvature coefficients) for determining what type of bifurcation (supercritical or subcritical) is going to take place in a degenerate Hopf bifurcation that involves the failure (vanishing) of some curvature coefficients. The approach takes advantage of the higher-order harmonic balance approximations, to obtain computational formulas for these coefficients. An application of these formulas will be given to three well-known examples: the van del Pol equation, a classical quadratic system, and a polynomic system with only cubic terms.

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Houston, Spring of 1996

Contents